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ANALYSIS OF INFLATION ON INVENTORY MODEL WITH NON INSTANTANEOUS DECAYING ITEMS

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ABSTRACT: An inventory is the amount of goods or materials contained in a store or factory at any given time for the purpose of future sale or production at the minimum cost of funds. Inventory generally means raw materials, semi finished, finished products, spares etc. stocked in order to meet future demand. Though inventory of materials is an ideal resource it is not meant for immediate use. It costs money in terms of storage, space, insurance, deterioration, equipment, personnel and above all the cost of capital involved in financing stock. Some inventories must be maintained for smooth and efficient functioning of an enterprise. Stocking of goods depends upon many factors such as demand, time lag, time of ordering etc. The fundamental reason for carrying inventories is that it is physically impossible and economically impractical for each stock item to arrive exactly where it is needed exactly when it is needed. The goal of inventory management is to ensure the consistent delivery of the right product in the right quantity to the right place at the right time

KEYWORDS: Inventory management, product, Shortage,

INTRODUCTION

As inventory represents a very important part of the company’s financial assets, it is very much affected by the market’s response to various situations, especially inflation. Inflation is a global phenomenon in present day times. Inflation can be defined as that state of disequilibrium in which an expansion of purchasing power tends to cause or is the effect of an increase in the price level. A period of prolonged, persistent and continuous inflation results in the economic, political, social and moral disruption of society. Almost everyone thinks inflation is evil, but it isn't necessarily so. Inflation affects different people in different ways. It also depends on whether inflation is anticipated or unanticipated. If the inflation rate corresponds to what the majority of people are expecting (anticipated inflation), then we can compensate, and the cost isn't high. Nowadays inflation has become a permanent feature in the inventory system. Inflation enters in the picture of inventory only because it may have an impact on the present value of the future inventory cost. Thus the inflation plays a vital role in the inventory system and production management though the decision makers may face difficulties in arriving at answers related to decision making. At present, it is impossible to ignore the effects of inflation and it is necessary to consider the effects of inflation on the inventory system.

REVIEW OF LITERATURE

Chang (2004) proposed an inventory model under a situation in which the supplier has provided a permissible delay in payments to the purchaser if the ordering quantity is greater than or equal to a predetermined quantity. Shortage was not allowed and the effect of the inflation rate, deterioration rate and delay in payments were discussed as well. Models for ameliorating/deteriorating items with time-varying demand pattern over a finite planning horizon were proposed by Moon et al. (2005). The effects of inflation and time value of money were also taken into account. Jolai et al. (2006) presented an optimization framework to derive optimal production over a fixed planning horizon for items with a stock-dependent demand rate under inflationary conditions. Jaggi et al. (2007) presented the optimal inventory replenishment policy for deteriorating items under inflationary conditions using a discounted cash flow (DCF) approach over a finite time horizon. Shortages in inventory were allowed and completely backlogged and demand rate was assumed to be a function of inflation. Chern, M. S. et al. (2008) developed an inventory lot-size model for deteriorating items with partial backlogging and time value of money. Roy, A., Pal, S. and Maiti, M.K. (2009) considered a production inventory model with inflation and time value of money and demand of the item is displayed stock dependent and lifetime of the product is random in nature and follows exponential distribution with a known mean. Yang, H.L., Teng, J.T. and Chern, M.S. (2010) developed an economic order quantity model under inflation with shortages.

ASSUMPTIONS

1. The product life time (time to deterioration) t has a p.d.f. $f(t)=\theta e^{-\theta(t-t_d)}$ for $t>t_d$. So that the deterioration rate is $r(t)=\dfrac{f(t)}{1-F(t)}=\theta$, for $t>t_d$.
2. The demand rate $D(t)=\alpha+\beta t$, where α and β are positive constants.
3. The effect of inflation is taken.
4. The replenishment rate is finite.
5. Lead time is zero.
6. Shortages are allowed and partial backlogged.
7. There is no replacement or repair of deteriorated units during the period under consideration.

NOTATIONS

A Ordering cost per order

q_i The i[th] price breaking point i=0,1,2…..m, where $0<q_0<q_1...<q_m$

C_i The purchasing cost per unit dependent on order size, i=0,1,2….m, where $C_0>C_1>...>C_m>0$.

(C_1+ht) Linear holding cost

C_S Shortages cost per unit backordered per unit time

C_{LS} The cost of lost sales per unit

$e^{-\delta t}$ Backlogging rate

θ Parameter of the deterioration rate function

t_d The length of time in which the product has no deterioration

t_1 The length of time with positive stock

T The replenishment cycle time, where $T\ge t_1$

Q Order quantity per cycle

$I_1(t)$ The inventory level at time t $(0\le t\le t_d)$ in which the product has no Deterioration

$I_2(t)$ The inventory level at time t $(t_d\le t\le T)$ in which the product has deterioration

$I_3(t)$ The inventory level at time t $(t_1\le t\le T)$ in which the product has Shortage

$I(t)$ The inventory level at time t $(0\le t\le T)$

$TVC_i(t_1,T)$ The total relevant inventory cost per unit of inventory system under the unit purchasing cost C_i

MODEL FORMULATION AND SOLUTION

This paper is developed the replenishment problem of a single non-instantaneous deteriorating item with partial backlogging and variable holding cost. $I_{\max}$ units of the item arrive at the inventory system at the beginning of each cycle. At, $[0,t_d]$ the inventory level decreases only owing to stock and time dependent demand rate. The inventory level drops to zero due to demand and deterioration of items at $[t_d,t_1]$. The shortage occurs and keeps to the end of the current order cycle. The entire process is repeated.

CASE I: When shortages are not allowed

The inventory level decreases only due to demand during the interval $[t_d,t_1]$. Hence the differential equation representing the inventory level is given by:

$$\frac{dI_1(t)}{dt} = -(\alpha+\beta t) \qquad 0 \le t \le t_d \qquad \ldots(1.1)$$

With the boundary condition $I_1(0)=I_{\max}$. Solution of equation (5.1) is:

$$I_1(t) = I_{\max} - (\alpha t + \frac{\beta t^2}{2}) \qquad 0 \le t \le t_d \qquad \ldots(1.2)$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -(\alpha+\beta t) \qquad t_d \le t \le t_1 \qquad \ldots(1.3)$$

With the boundary condition $I_2(t_1)=0$. Solution of equation (5.3) is:

$$I_2(t) = [\frac{\alpha}{\theta}(e^{\theta(t_1-t)}-1) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1-t)} - t) - \frac{\beta}{\theta^2}(e^{\theta(t_1-t)}-1)] \quad t_d \le t \le t_1 \qquad \ldots(1.4)$$

Equating the equation (5.2) and (5.4) at $t=t_d$

$$I_{\max} - (\alpha t_d + \frac{\beta t_d{}^2}{2}) = [\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1-t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)]$$

This implies that the maximum inventory level for each cycle is:

$$I_{\max} = [\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1-t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1) - (\alpha t_d + \frac{\beta t_d{}^2}{2})] \qquad \ldots(1.5)$$

Substitute the value of $I_{\max}$ from equation (1.5) in equation (1.2), we get

$$I_1(t) = [\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1-t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1) - (\alpha t_d + \frac{\beta t_d{}^2}{2})] - (\alpha t + \frac{\beta t^2}{2})$$

$$0 \le t \le t_d \qquad \ldots(1.6)$$

From equation (5.5), we can obtain the order quantity Q as:

$$Q = I_{\max}$$

$$= [\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1-t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1) - (\alpha t_d + \frac{\beta t_d{}^2}{2})] \qquad \ldots(1.7)$$

The total relevant cost per unit time consists of

Ordering cost $O_C = \frac{A}{T}$ $\ldots(1.8)$

Inventory holding cost $H_C = \frac{1}{T}[\int_0^{t_d} C_1 I(t) e^{-rt} dt + \int_{t_d}^{t_1} C_1 I(t) e^{-rt} dt]$

$$= \frac{1}{T}[C_1\{\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)} - t_d)t_d + \frac{\beta}{\theta}(t_1 e^{\theta(t_1-t_d)} - t_d)t_d - \frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)t_d$$

$$-(\frac{3\alpha t_d^{\,2}}{2}+\frac{2\beta t_d^{\,3}}{3})-\frac{r\alpha t_d^{\,2}}{2\theta}(e^{\theta(t_1-t_d)}-t_d)-\frac{r\beta t_d^{\,2}}{2\theta}(t_1e^{\theta(t_1-t_d)}-t_d)$$
$$-\frac{r\beta t_d^{\,2}}{2\theta^2}(e^{\theta(t_1-t_d)}-1)-\frac{rt_d^{\,2}}{2}(\alpha t_d+\frac{\beta t_d^{\,2}}{2})+r(\frac{\alpha t_d^{\,3}}{3}+\frac{\beta t_d^{\,4}}{8})\}$$
$$+\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})$$
$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\,2}}{\theta}+\frac{t_1t_de^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,3}}{3}+\frac{t_d^{\,3}}{3})$$
$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})\}] \qquad \ldots(1.9)$$

Inventory deterioration cost $D_C=\frac{C_d}{T}[\int_{t_d}^{t_1}\theta I(t)e^{-rt}dt]$

$$=\frac{C_d\theta}{T}[\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})$$
$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\,2}}{\theta}+\frac{t_1t_de^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,3}}{3}+\frac{t_d^{\,3}}{3})$$
$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})\}] \qquad \ldots(1.10)$$

Purchase cost $P_C=C_iQ$

$$=C_i[\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)+\frac{\beta}{\theta}(t_1e^{\theta(t_1-t_d)}-t_d)-\frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)-(\alpha t_d+\frac{\beta t_d^{\,2}}{2})] \qquad \ldots(1.11)$$

The total relevant inventory cost per unit time can be formulated as:

$TVC(t_1,T)=O_C+H_C+D_C+P_C$

$$=\frac{A}{T}+\frac{1}{T}[C_1\{\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)t_d+\frac{\beta}{\theta}(t_1e^{\theta(t_1-t_d)}-t_d)t_d-\frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)t_d$$
$$-(\frac{3\alpha t_d^{\,2}}{2}+\frac{2\beta t_d^{\,3}}{3})-\frac{r\alpha t_d^{\,2}}{2\theta}(e^{\theta(t_1-t_d)}-t_d)-\frac{r\beta t_d^{\,2}}{2\theta}(t_1e^{\theta(t_1-t_d)}-t_d)$$
$$-\frac{r\beta t_d^{\,2}}{2\theta^2}(e^{\theta(t_1-t_d)}-1)-\frac{rt_d^{\,2}}{2}(\alpha t_d+\frac{\beta t_d^{\,2}}{2})+r(\frac{\alpha t_d^{\,3}}{3}+\frac{\beta t_d^{\,4}}{8})\}$$
$$+\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})$$
$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\,2}}{\theta}+\frac{t_1t_de^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,3}}{3}+\frac{t_d^{\,3}}{3})$$

$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_d e^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})\}]$$
$$+\frac{C_d\theta}{T}[\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1 e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})$$
$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_d e^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\,2}}{\theta}+\frac{t_1 t_d e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1 e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,3}}{3}+\frac{t_d^{\,3}}{3})$$
$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_d e^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})\}]$$
$$+C_i[\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)+\frac{\beta}{\theta}(t_1 e^{\theta(t_1-t_d)}-t_d)-\frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)-(\alpha t_d+\frac{\beta t_d^{\,2}}{2})] \quad \ldots(1.12)$$

To minimize the total average cost per unit of time, the optimal value of t_1 and T can be obtained by the following equations

$$\frac{\partial TVC(t_1,T)}{\partial t_1}=0 \quad \text{and} \quad \frac{\partial TVC(t_1,T)}{\partial T}=0$$

$$\frac{\partial^2 TVC(t_1,T)}{\partial^2 t_1}>0 \quad \text{and} \quad \frac{\partial^2 TVC(t_1,T)}{\partial^2 T}>0$$

SPECIAL CASES

CASE A: When $\beta=0$**,** demand rate is constant. Then the total relevant inventory cost per unit time can be formulated as:

$$TVC(t_1,T)=O_C+H_C+D_C+P_C$$
$$=\frac{A}{T}+\frac{1}{T}[C_1\{\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)t_d-\frac{3\alpha t_d^{\,2}}{2}-\frac{r\alpha t_d^{\,2}}{2\theta}(e^{\theta(t_1-t_d)}-t_d)-\frac{r\alpha t_d^{\,3}}{6}\}$$
$$+\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_d e^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})\}]+\frac{C_d\theta}{T}[\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))$$
$$-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_d e^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})\}]$$
$$+C_i[\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)-(\alpha t_d)]$$

CASE I: When shortages are partially backlogged

The inventory level decreases only due to demand during the interval $[t_d,t_1]$. Hence the differential equation representing the inventory level is given by:

$$\frac{dI_1(t)}{dt}=-(\alpha+\beta t) \qquad 0\le t\le t_d \qquad \ldots(1.13)$$

With the boundary condition $I_1(0)=I_{\max}$. Solution of equation (1.13) is:

$$I_1(t) = I_{\max} - (\alpha t + \frac{\beta t^2}{2}) \qquad 0 \le t \le t_d \qquad \ldots(1.14)$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -(\alpha + \beta t) \qquad t_d \le t \le t_1 \qquad \ldots(1.15)$$

With the boundary condition $I_2(t_1) = 0$. Solution of equation (1.15) is:

$$I_2(t) = [\frac{\alpha}{\theta}(e^{\theta(t_1 - t)} - 1) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1 - t)} - t) - \frac{\beta}{\theta^2}(e^{\theta(t_1 - t)} - 1)] \qquad t_d \le t \le t_1 \qquad \ldots(1.16)$$

Equating the equation (1.14) and (1.16) at $t = t_d$

$$I_{\max} - (\alpha t_d + \frac{\beta t_d^{\ 2}}{2}) = [\frac{\alpha}{\theta}(e^{\theta(t_1 - t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1 - t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1 - t_d)} - 1)]$$

This implies that the maximum inventory level for each cycle is:

$$I_{\max} = [\frac{\alpha}{\theta}(e^{\theta(t_1 - t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1 - t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1 - t_d)} - 1) - (\alpha t_d + \frac{\beta t_d^{\ 2}}{2})] \qquad \ldots(1.17)$$

Substitute the value of I_{max} from equation (1.17) in equation (1.14), we get

$$I_1(t) = [\frac{\alpha}{\theta}(e^{\theta(t_1 - t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1 - t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1 - t_d)} - 1) - (\alpha t_d + \frac{\beta t_d^{\ 2}}{2})] - (\alpha t + \frac{\beta t^2}{2})$$

$$0 \le t \le t_d \qquad \ldots (1.18)$$

During the shortage interval $[t_1, T]$, the demand at any time t is partially backlogged at fraction. Thus the inventory level at any time t is governed by the following equation:

$$\frac{dI_3(t)}{dt} = -(\alpha + \beta t)e^{-\delta t} \qquad t_1 \le t \le T \qquad \ldots(1.19)$$

$$I_3(t) = [\frac{\alpha}{\delta}(e^{-\delta t} - e^{-\delta t_1}) + \frac{\beta}{\delta}(te^{-\delta t} - t_1 e^{-\delta t_1}) - \frac{\beta}{\delta^2}(e^{-\delta t} - e^{-\delta t_1})] \qquad \ldots(1.20)$$

Substitute t=T in equation (1.20), we obtain the maximum amount of demand backlogged per cycle as follows

$$S = (-I_3(t))$$

$$= [\frac{\alpha}{\delta}(e^{-\delta t_1} - e^{-\delta T}) - \frac{\beta}{\delta}(Te^{-\delta T} - t_1 e^{-\delta t_1}) + \frac{\beta}{\delta^2}(e^{-\delta T} - e^{-\delta t_1})] \qquad \ldots(1.21)$$

From equation (1.17), we can obtain the order quantity Q as:

$$Q = I_{\max} + S$$

$$= [\frac{\alpha}{\theta}(e^{\theta(t_1 - t_d)} - t_d) + \frac{\beta}{\theta}(t_1 e^{\theta(t_1 - t_d)} - t_d) - \frac{\beta}{\theta^2}(e^{\theta(t_1 - t_d)} - 1) - (\alpha t_d + \frac{\beta t_d^{\ 2}}{2})$$

$$+ \frac{\alpha}{\delta}(e^{-\delta t_1} - e^{-\delta T}) - \frac{\beta}{\delta}(Te^{-\delta T} - t_1 e^{-\delta t_1}) + \frac{\beta}{\delta^2}(e^{-\delta T} - e^{-\delta t_1})] \qquad \ldots(1.22)$$

The total relevant cost per unit time consists of

Ordering cost $O_C = \frac{A}{T}$ $\qquad \ldots(1.23)$

Inventory holding cost $H_C = \frac{1}{T}[\int_0^{t_d} C_1 I(t) e^{-rt} dt + \int_{t_d}^{t_1} C_1 I(t) e^{-rt} dt]$

$$=\frac{1}{T}[C_1\{\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)t_d+\frac{\beta}{\theta}(t_1e^{\theta(t_1-t_d)}-t_d)t_d-\frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)t_d$$
$$-(\frac{3\alpha t_d^{\ 2}}{2}+\frac{2\beta t_d^{\ 3}}{3})-\frac{r\alpha t_d^{\ 2}}{2\theta}(e^{\theta(t_1-t_d)}-t_d)-\frac{r\beta t_d^{\ 2}}{2\theta}(t_1e^{\theta(t_1-t_d)}-t_d)$$
$$-\frac{r\beta t_d^{\ 2}}{2\theta^2}(e^{\theta(t_1-t_d)}-1)-\frac{rt_d^{\ 2}}{2}(\alpha t_d+\frac{\beta t_d^{\ 2}}{2})+r(\frac{\alpha t_d^{\ 3}}{3}+\frac{\beta t_d^{\ 4}}{8})\}$$
$$+\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\ 2}}{2}+\frac{t_d^{\ 2}}{2})$$
$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\ 2}}{2}+\frac{t_d^{\ 2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\ 2}}{\theta}+\frac{t_1t_de^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\ 3}}{3}+\frac{t_d^{\ 3}}{3})$$
$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\ 2}}{2}+\frac{t_d^{\ 2}}{2})\}] \qquad \ldots(1.24)$$

Inventory deterioration cost $D_C=\frac{C_d}{T}[\int_{t_d}^{t_1}\theta I(t)e^{-rt}dt]$

$$=\frac{C_d\theta}{T}[\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\ 2}}{2}+\frac{t_d^{\ 2}}{2})$$
$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\ 2}}{2}+\frac{t_d^{\ 2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\ 2}}{\theta}+\frac{t_1t_de^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\ 3}}{3}+\frac{t_d^{\ 3}}{3})$$
$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\ 2}}{2}+\frac{t_d^{\ 2}}{2})\}] \qquad \ldots(1.25)$$

Purchase cost $P_C=C_iQ$

$$=C_i[\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)+\frac{\beta}{\theta}(t_1e^{\theta(t_1-t_d)}-t_d)-\frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)-(\alpha t_d+\frac{\beta t_d^{\ 2}}{2})$$
$$+\frac{\alpha}{\delta}(e^{-\delta t_1}-e^{-\delta T})-\frac{\beta}{\delta}(Te^{-\delta T}-t_1e^{-\delta t_1})+\frac{\beta}{\delta^2}(e^{-\delta T}-e^{-\delta t_1})] \qquad \ldots(1.26)$$

Shortages cost $S_C=\frac{C_S}{T}[\int_{t_1}^{T}(-I(t))e^{-rt}dt]$

$$=\frac{C_S}{T}[(\frac{\alpha}{\delta}-\frac{\beta}{\delta^2})(e^{-\delta t_1}(T-t_1)+\frac{e^{-\delta T}}{\delta}-\frac{e^{-\delta t_1}}{\delta})+\frac{\beta}{\delta}(t_1e^{-\delta t_1}(T-t_1)+\frac{Te^{-\delta T}}{\delta}$$
$$-\frac{t_1e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})-\frac{\beta}{\delta^2}(e^{-\delta t_1}(T-t_1)+\frac{e^{-\delta T}}{\delta}-\frac{e^{-\delta t_1}}{\delta})$$
$$-\frac{\alpha r}{\delta}(\frac{(T^2-t_1^{\ 2})e^{-\delta t_1}}{2}+\frac{Te^{-\delta T}}{\delta}-\frac{t_1e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})$$
$$-\frac{\beta r}{\delta}(\frac{(T^2-t_1^{\ 2})t_1e^{-\delta t_1}}{2}+\frac{(T^2e^{-\delta T}-t_1^{\ 2}e^{-\delta t_1})}{\delta}+\frac{2}{\delta}(\frac{Te^{-\delta T}}{\delta}-\frac{t_1e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})$$

$$+\frac{r\beta}{\delta}(\frac{(T^2-t_1^{\,2})e^{-\delta t_1}}{2}+\frac{Te^{-\delta T}}{\delta}-\frac{t_1e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})] \quad \ldots(1.27)$$

Lost sale cost $L_C=\frac{C_{LS}}{T}[\int_{t_1}^{T}(1-e^{-\delta t})(\alpha+\beta t)e^{-rt}dt]$

$$=\frac{C_{LS}}{T}[\alpha(-\frac{e^{-rT}}{r}+\frac{e^{-rt_1}}{r}+\frac{e^{-(r+\delta)T}}{(r+\delta)}-\frac{e^{-(r+\delta)t_1}}{(r+\delta)})+\beta(-\frac{Te^{-rT}}{r}+\frac{t_1e^{-rt_1}}{r}$$
$$-\frac{e^{-rT}}{r^2}+\frac{e^{-rt_1}}{r^2}+\frac{Te^{-(r+\delta)T}}{(r+\delta)}-\frac{t_1e^{-(r+\delta)t_1}}{(r+\delta)}-\frac{e^{-(r+\delta)T}}{(r+\delta)^2}+\frac{e^{-(r+\delta)t_1}}{(r+\delta)^2}] \quad \ldots(1.28)$$

The total relevant inventory cost per unit time can be formulated as:

$TVC(t_1,T)=O_C+H_C+D_C+P_C+S_C+L_C$

$$=\frac{A}{T}+\frac{1}{T}[C_1\{\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)t_d+\frac{\beta}{\theta}(t_1e^{\theta(t_1-t_d)}-t_d)t_d-\frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)t_d$$
$$-(\frac{3\alpha t_d^{\,2}}{2}+\frac{2\beta t_d^{\,3}}{3})-\frac{r\alpha t_d^{\,2}}{2\theta}(e^{\theta(t_1-t_d)}-t_d)-\frac{r\beta t_d^{\,2}}{2\theta}(t_1e^{\theta(t_1-t_d)}-t_d)$$
$$-\frac{r\beta t_d^{\,2}}{2\theta^2}(e^{\theta(t_1-t_d)}-1)-\frac{rt_d^{\,2}}{2}(\alpha t_d+\frac{\beta t_d^{\,2}}{2})+r(\frac{\alpha t_d^{\,3}}{3}+\frac{\beta t_d^{\,4}}{8})\}$$
$$+\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})$$
$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\,2}}{\theta}+\frac{t_1t_de^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,3}}{3}+\frac{t_d^{\,3}}{3})$$
$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})\}]$$
$$+\frac{C_d\theta}{T}[\{\frac{\alpha}{\theta}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))+\frac{\beta}{\theta}(-\frac{t_1}{\theta}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})$$
$$-\frac{\beta}{\theta^2}(-\frac{1}{\theta}+\frac{e^{\theta(t_1-t_d)}}{\theta}-(t_1-t_d))-\frac{r\alpha}{\theta}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}$$
$$-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})-\frac{r\beta}{\theta}(-\frac{t_1^{\,2}}{\theta}+\frac{t_1t_de^{\theta(t_1-t_d)}}{\theta}-\frac{t_1}{\theta^2}+\frac{t_1e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,3}}{3}+\frac{t_d^{\,3}}{3})$$
$$+\frac{r\beta}{\theta^2}(-\frac{t_1}{\theta}+\frac{t_de^{\theta(t_1-t_d)}}{\theta}-\frac{1}{\theta^2}+\frac{e^{\theta(t_1-t_d)}}{\theta^2}-\frac{t_1^{\,2}}{2}+\frac{t_d^{\,2}}{2})\}]$$
$$+C_i[\frac{\alpha}{\theta}(e^{\theta(t_1-t_d)}-t_d)+\frac{\beta}{\theta}(t_1e^{\theta(t_1-t_d)}-t_d)-\frac{\beta}{\theta^2}(e^{\theta(t_1-t_d)}-1)-(\alpha t_d+\frac{\beta t_d^{\,2}}{2})$$
$$+\frac{\alpha}{\delta}(e^{-\delta t_1}-e^{-\delta T})-\frac{\beta}{\delta}(Te^{-\delta T}-t_1e^{-\delta t_1})+\frac{\beta}{\delta^2}(e^{-\delta T}-e^{-\delta t_1})]$$
$$+\frac{C_S}{T}[(\frac{\alpha}{\delta}-\frac{\beta}{\delta^2})(e^{-\delta t_1}(T-t_1)+\frac{e^{-\delta T}}{\delta}-\frac{e^{-\delta t_1}}{\delta})+\frac{\beta}{\delta}(t_1e^{-\delta t_1}(T-t_1)+\frac{Te^{-\delta T}}{\delta}$$

$$-\frac{t_1 e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})-\frac{\beta}{\delta^2}(e^{-\delta t_1}(T-t_1)+\frac{e^{-\delta T}}{\delta}-\frac{e^{-\delta t_1}}{\delta})$$
$$-\frac{\alpha r}{\delta}(\frac{(T^2-t_1{}^2)e^{-\delta t_1}}{2}+\frac{Te^{-\delta T}}{\delta}-\frac{t_1 e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})$$
$$-\frac{\beta r}{\delta}(\frac{(T^2-t_1{}^2)t_1 e^{-\delta t_1}}{2}+\frac{(T^2 e^{-\delta T}-t_1{}^2 e^{-\delta t_1})}{\delta}+\frac{2}{\delta}(\frac{Te^{-\delta T}}{\delta}-\frac{t_1 e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})$$
$$+\frac{r\beta}{\delta}(\frac{(T^2-t_1{}^2)e^{-\delta t_1}}{2}+\frac{Te^{-\delta T}}{\delta}-\frac{t_1 e^{-\delta t_1}}{\delta}+\frac{e^{-\delta T}}{\delta^2}-\frac{e^{-\delta t_1}}{\delta^2})]$$
$$+\frac{C_{LS}}{T}[\alpha(-\frac{e^{-rT}}{r}+\frac{e^{-rt_1}}{r}+\frac{e^{-(r+\delta)T}}{(r+\delta)}-\frac{e^{-(r+\delta)t_1}}{(r+\delta)})+\beta(-\frac{Te^{-rT}}{r}+\frac{t_1 e^{-rt_1}}{r}$$
$$-\frac{e^{-rT}}{r^2}+\frac{e^{-rt_1}}{r^2}+\frac{Te^{-(r+\delta)T}}{(r+\delta)}-\frac{t_1 e^{-(r+\delta)t_1}}{(r+\delta)}-\frac{e^{-(r+\delta)T}}{(r+\delta)^2}+\frac{e^{-(r+\delta)t_1}}{(r+\delta)^2}] \qquad \ldots(1.29)$$

To minimize the total average cost per unit of time, the optimal value of t_1 and T can be obtained by the following equations

$$\frac{\partial TVC(t_1,T)}{\partial t_1}=0 \quad \text{and} \quad \frac{\partial TVC(t_1,T)}{\partial T}=0$$

$$\frac{\partial^2 TVC(t_1,T)}{\partial^2 t_1}>0 \quad \text{and} \quad \frac{\partial^2 TVC(t_1,T)}{\partial^2 T}>0$$

CONCLUSION

An inventory model of determining the optimal replenishment policy for non-instantaneous deteriorating items with linear demand has been developed. In many cases customers are conditioned to a shipping delay, and may be willing to wait for a short time in order to get their first choice. Generally speaking, the length of the waiting time for the next replenishment is the main factor for deciding whether the backlogging will be accepted or not. The willingness of a customer to wait for backlogging during a shortage period declines with the length of the waiting time. The main goal of this study is to minimize the total average cost. The model developed may further be extended for production inventory model, inflation and permissible delay in payments.

REFERENCES

1. Ben-Daya, M. (2014): Optimal time-varying lot sizing models under inflationary conditions. *E.J.O.R., 89 (2), 313-325.*
2. Chang, C.T. (2004): An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity. I.J.P.E., 88, 307-311.
3. Chaudhuri, K.S. (2005): An EOQ model with stock-dependent demand, shortage, inflation and time discounting. *I.J.P.E., 53 (2), 171-180.*
4. Chen. J.M. (1998): An inventory model for deteriorating items with time-proportional demand and shortages under inflation and time discounting. I.J.P.E., 55 (1), 21-30.
5. Bansal K.K.(20130: Inventory Model for Deteriorating Items with the Effect of inflation, International Journal of Application or Innovation in Engineering & Management (IJAIEM) 143-150
6. Anand, Bansal K.K. (2013) : An Optimal Production Model or Deteriorating Item WithStocks and Price Sensitive Demand Rate, Journal of Engineering, Computers & Applied Sciences (JEC&AS), p.p. 32-37
7. Chern, M.S. (2010). An inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages. Int. J. Production Economics, 123, 8–19.
8. Goel, S.K. (2011): Optimal order policy for deteriorating items with inflation induced demand. *I.J.P.E., 103 (2), 707-714.*
9. Hariga. M.A. (1995): Effects of inflation and time-value of money on an inventory model on an inventory model with time-dependent demand rate and shortages. *E.J.O.R., 81 (3), 512-520.*

10. Ko, B. (2010): Economic order quantity models for ameliorating / deteriorating items under inflation and time discounting. *E.J.O.R., 162 (3), 773-785.*
11. Lee, S. (2012): The effects of inflation and time-value of money on an economic order quantity model with a random product life cycle. *E.J.O.R., 125, 588-601.*
12. Maiti, M.K. (2009). A production inventory model with stock dependent demand incorporating learning and inflationary effect in a random planning horizon: A fuzzy genetic algorithm with varying population size approach. *Computers & Industrial Engineering, 57, 1324–1335.*
13. Misra, R.B. (1975): A study of inflationary effects on inventory systems. *Logistic Spectrum*, 9(3), 260-268.
14. Misra, R.B. (1979): A note on optimal inventory management under inflation. N.R.L., 26, 161-165.
15. Sadoughian, M.R. (2012): An economic production lot size model with deteriorating items, stock-dependent demand, inflation, and partial backlogging. Applied Mathematics and Computation (A.M.C.), 181, 380-389.
16. Yang, H. L., Teng, J.T, and Papachristos, S. (2012). Partial backlogging inventory lot-size models for deteriorating items with fluctuating demand under inflation. *European Journal of Operational Research, 191, 127–141.*
17. Yang, H.L. (2004): Two-warehouse inventory models for deteriorating items with shortages under inflation. E.J.O.R., 157, 344-351.